

For a homogeneous nuclear reactor producing a constant power, the following holds:

$$\frac{d}{dt} \log P = -\frac{\alpha}{\tau} T$$

$$\frac{d}{dt} T = (P-1) \frac{1}{\epsilon}$$

where

$P(t)$  = instantaneous reactor power

$-\alpha$  = temperature coefficient

$\tau$  = half time of a neutron

$\epsilon$  = heat capacity

To obtain the canonical form, use the transformation

$$x_1 = \log P \text{ and } x_2 = -\frac{\alpha}{\tau} T$$

This leads to the system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{\alpha}{\epsilon \tau} \{e^{x_1} - 1\}$$

The Lyapunov function of this system is

$$V(\mathbf{x}) = \frac{1}{2} x_2^2 + \frac{\alpha}{\epsilon \tau} \{e^{x_1} - x_1\} - \frac{\alpha}{\epsilon \tau}$$

$$\dot{V}(\mathbf{x}) = -\left(\frac{\alpha}{\tau}\right)^2 \frac{T}{\epsilon} \{P-1\} < 0 \text{ if } P > 1 \text{ and } \dot{V} > 0 \text{ if } P < 1$$

$V(x)$  is positive definite so the equilibrium point  $P = 1$ ,  $T = 0$  is stable as the three parameters  $\alpha$ ,  $\tau$  and  $\epsilon$  are positive.

If  $P$  is less than 1, the reactor is unstable. Cooling is essential.

A well known fact.